Holdup and Pressure Drop with Gas-Liquid Flow in a Vertical Pipe

G. A. HUGHMARK and B. S. PRESSBURG

Louisiana State University, Baton Rouge, Louisiana

Vertical upward concurrent air-liquid flow was investigated under isothermal conditions in a test section of 1-in. schedule 40 pipe. Pressure drop was measured with a mercury manometer connected to two pressure taps 20 ft. apart in the section. Liquid was trapped between two quick shutoff valves activated by two solenoid valves. The liquid was drained from the section to provide the holdup data. Six liquids were used to determine the effect of density, viscosity, and surface tension.

The experimental holdup, and two-phase pressure drop data were not in agreement with the **Lockhart-Martinelli type of correlation for horizontal flow. A statistical correlation for holdup was developed to include fluid physical properties, total mass velocity, and the air-liquid ratio entering the pipe. Similarly a pressure drop correlation was developed which expressed the two-phase pressure drop as a function of the slip velocity, liquid physical properties, and total mass velocity. This correlation showed an average percentage error of less than 15% between the observed and the calculated total pressure drop.**

Air-water two-phase flow was utilized as early as 1797 in an air-lift pump invented by a German, Carl Loscher. Vertical two-phase flow was again used in 1927 in the Seminole, Oklahoma, oil fields to produce oil by gas lift. Cromer (*2*) in 1936 visually observed the upward flow of mixtures of air and water in a vertical pipe. Four flow regimes were reported, but in practice the transition between regimes is indistinct and it is sometimes difficult to determine the regime for a given condition. Schurig (*17*), Kegel (*10*), Dukler (*3*), McElwee (*14*), Galegar, Stovall, and Huntington (*5*), and Govier, Radford, and Dunn (*6*) have obtained data for the air-water system in vertical upward flow. Yagi and co-workers (*21, 22*) obtained data for the air-water and air-oil systems over limited ranges of flow. Schwarz (*18*) reported holdup data for the steam-water system at 20- to 80-atm. pressure. Isbin, Sher, and Eddy (*7*) reported holdup data for the steam-water system at essentially atmospheric pressure. No satisfactory correlation method is presented for either holdup or pressure drop in isothermal two-phase flow based on these data for pipe diameters of 0.5 to 2.4 in.

Lockhart and Martinelli (*11*) in 1949 presented a correlation that enabled the prediction of pressure drops for horizontal flow to within about plus or minus 30%. A holdup correlation was also presented. Later data by Jenkins (*8*) indicated that mass velocity is an important variable that is not included in the Lockhart-Martinelli correlation. Chenoweth and Martin (*1*) in 1955 published a correlation for pressure drop in horizontal turbulent flow. Experimental data were obtained in 1-½-in. and 3-in. pipes at pressures up to 100 lb./sq. in. abs. The correlation shows agreement within 20% for these data and the data used by Lockhart and Martinelli for their correlation.

Vertical steam-water flow at high pressure has been of interest for nuclear reactors. Egen, Dingee, and Chastain (*4*) reported holdup data for steam-water at 2,000 lb./sq. in. abs. in a channel 1 in. by 0.103 in. cross section. Studies of pressure drop and heat transfer during boiling have been reported by many investigators (*9, 12, 13, 19, 20*).

The purpose of the research described here was to obtain data for a variety of liquids over a wide range of flow rates and from this to obtain generalized correlations that would permit reliable prediction of holdup and pressure drop in vertical upward flow under isothermal conditions.

EXPERIMENTAL EQUIPMENT AND METHOD OF OPERATION

The vertical test section was a 24.2 ft. long run of 1-in. schedule 40 steel pipe with pressure taps 20 ft. apart. The bottom pressure tap was 28 in. above an air-liquid mixing nozzle. Surge pots 3 in. in diameter by 6 in. long were installed with the tap from the test section entering the side of the pot. The outlet from the bottom of each pot was connected with copper tubing to a 30-in differential manometer which used mercury as the metering element. The portion of the pots below the taps and the copper tubing were filled with the test liquid as a seal fluid on the mercury. The diameter of the surge pots was selected so that the cross-sectional area of the pots was about 200 times the cross-sectional area of the manometer. A change in manometer level thus reflected a negligible change in level in the surge pot.

One quick shutoff valve was installed below the bottom pressure tap and a second above the top pressure tap. These 1-in. valves were closed by air pressure from two solenoid valves. Each solenoid and shutoff valve was connected by identical piping, and the solenoids were operated from a single electrical switch to obtain simultaneous shutoff of the valves.

A pressure gauge was installed in the test section at the elevation of the bottom pressure tap. A drain connection was provided above the bottom valve, and a vent was installed below the top valve.

The desired liquid flow for a run was metered to the test section through a rotameter. After the manometer had reached a steady reading, the desired gas flow was metered with a second rotame-

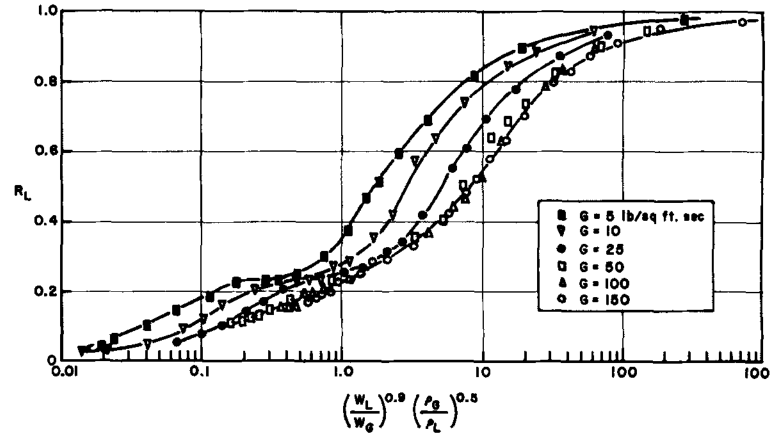

Fig. 1. Holdup vs. $(W_L/W_G)^{0.9}\ (\rho_G/\rho_L)^{0.5}$ for water.

G. A. Hughmark is with Ethyl Corporation, Baton Rouge, Louisiana.

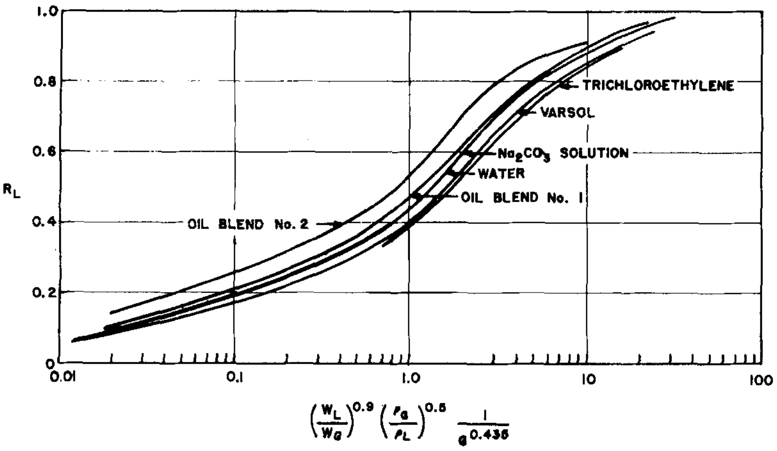

Fig. 2. Holdup vs. $(W_L/W_G)^{0.9}$ $(\rho_G/\rho_L)^{0.5}$ $1/G^{0.435}$ for all liquids.

ter. Steady state of most of the two-phase runs was obtained in less than 1 min. Runs at high liquid rates and very low air rates required several minutes to reach steady state conditions as indicated by constancy of the manometer reading. The rotameter readings, manometer reading, and pressure gauge reading were recorded for each run. The valves were closed, the vent was opened, and the liquid was drained from the test section to a graduated cylinder. The volume of liquid drained from the test section was recorded.

Runs were made with air and six different liquids: water, a saturated aqueous solution of sodium carbonate, kerosene, trichloroethylene, and two oils with viscosities of 5.8 and 28.6 centipoises. These liquids were selected to vary the physical properties of density, viscosity, and surface tension. Because of the extremely turbulent nature of two-phase flow, dynamic rather than static surface tension would be expected to be an important variable. Therefore use of surface active agents was purposely avoided in this investigation.

The experimental data for holdup and pressure drop represent an average value for the 20-ft. section of pipe. Pressure change is essentially linear for the length of the pipe. Therefore results from each run were assumed to represent a point value at the mid-point of the section. The vapor volume for the point condition then corresponds to the arithmetic mean of the inlet and outlet pressures for the pipe section.

HOLDUP CORRELATION

Lockhart and Martinelli used the factor

$$X = \left(\frac{W_L}{W_G}\right)^{0.9}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}\left(\frac{\mu_L}{\mu_G}\right)^{0.1}$$

to correlate the liquid holdup R_L for horizontal flow. When plotted against this factor the experimental data of this investigation showed a series of lines, each parallel to the Lockhart-Martinelli correlation. The magnitude of the deviation from the Lockhart-Martinelli line appeared to be a function of total mass velocity.

Table 1. Summary of Coefficients and Standard Deviations for Holdup Correlation—Experimental Data

Physical property	Coefficient	Standard deviation
ρ_L	—0.70989	0.265
μ_L	+0.18971	0.040
σ	+0.20448	0.106

The term $\left(\frac{W_L}{W_G}\right)^{0.9}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}$ was used as the initial correlating factor. This was plotted vs. R_L with a separate plot for each liquid. Figure 1 shows the plot for the air-water data. The plots for the other liquid were similar. Data for total mass velocities of 50, 100, and 150 lb./sq. ft. sec. were found to fall together. The function $1/G^{0.435}$ was found to correlate the parameters for total mass velocities of 5, 10, 25, and 50. A separate plot was then made for each liquid of R_L vs.

$$\frac{1}{G^{0.435}}\left(\frac{W_L}{W_G}\right)^{0.9}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}$$

Table 2. Summary of Coefficients and Standard Deviations for Holdup Correlation—Schwarz Data and Experimental Data

Physical property	Coefficient	Standard deviation
ρ_L	—0.7205	0.22
μ_L	+0.18773	0.032
σ	+0.2054	0.09
ρ_G	+0.70266	0.10
μ_G	+2.7539	0.56

Since, as observed in Figure 1, for values of G above 50 there was no perceptible variation in R_L with increasing G, a maximum value of 50 was used for G in evaluating this group; this was found to give better agreement between the observed data and the correlation than if the actual values of G were used.

The G in this correlating group is dimensional. Application of dimensional analysis to this problem generally yields the ratios W_L/W_G, ρ_L/ρ_G, μ_L/μ_G, and the Reynolds' number. Since D and μ_G were constant in this study, any variation in G causes a corresponding change in the Reynolds' number. G was used inasmuch as the present study does not provide evidence either for or against incorporating it in a Reynolds' number.

A best line was drawn for each fluid and these lines were then plotted as shown by Figure 2.

The lines are essentially parallel for the different liquids. At R_L = 0.60, the value of $\frac{1}{G^{0.435}}\left(\frac{W_L}{W_G}\right)^{0.9}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}$ was obtained for each liquid. This was divided by $\left(\frac{\rho_G}{\rho_L}\right)^{0.5}$ at the average conditions of the runs to eliminate density from the terms and thus obtain the best value of $\frac{1}{G^{0.435}}\left(\frac{W_L}{W_G}\right)^{0.9}$ for

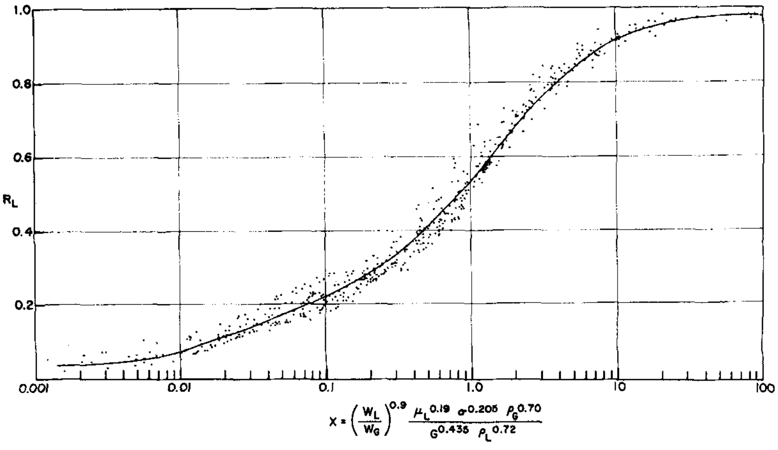

Fig. 3. Correlation of holdup data with x.

each liquid. This value was correlated by the equation

$$\log\left[\frac{1}{G^{0.435}}\left(\frac{W_L}{W_G}\right)^{0.9}\right] = \log a_0 + a_1 \log \rho_L + a_2 \log \mu_L + a_3 \log \sigma \qquad (1)$$

A least-squares calculation was made with a computer to obtain the best fit to the equation. The correlation coefficient for the least-squares fit was 0.98, which indicates a high level of significance of the fit of the equation to the data. The data obtained are shown in Table 1.

The coefficients are twice the standard deviation or higher. All three coefficients are therefore at the 95% confidence level or better.

Correlation of the experimental data did not consider either density or viscosity of the gas phase because these did not vary significantly. Schwarz (*18*) reports holdup data on water-steam at 20- to 80-atm pressure. These data were worked up in the same manner as the experimental data for correlation to obtain coefficients for ρ_G and μ_G. The results are shown by Table 2. The coefficients for the liquid physical properties are essentially the same as for the correlation of the experimental data alone. All coefficients are above the 98% confidence level. The correlation coefficient was 0.99.

The correlating function for R_L is thus found to be

$$\left(\frac{W_L}{W_G}\right)^{0.9} \frac{\mu_L^{0.19}\,\sigma^{0.205}\,\rho_G^{0.70}\,\mu_G^{2.75}}{G^{0.435}\,\rho_L^{0.72}}$$

where G is a maximum of 50 lb./sq. ft. sec. The experimental data were plotted with this function but with μ_G, which was constant at 0.018 centipoises, neglected. Gas viscosity is then added to the function by the factor

$$\left(\frac{\mu_G}{0.018}\right)^{2.75} = 6 \times 10^4 \mu_G^{2.75}$$

The final correlation shown by Figure 3 plots R_L against the function x:

$$x = 6 \times 10^4 \left(\frac{W_L}{W_G}\right)^{0.9} \frac{\mu_L^{0.19}\,\sigma^{0.205}\,\rho_G^{0.70}\,\mu_G^{2.75}}{G^{0.435}\,\rho_L^{0.72}} \qquad (2)$$

This proposed correlation was compared with the experimental data by determining $\Delta R_L = R_L$ (observed) − R_r (calculated).

The average absolute ΔR_L was 0.022 for the 551 runs. The correlation was also compared in the same way with holdup data reported by seven investigators with data on the air-water and steam-water systems with tube diameters from 0.4 to 2.34 in. The average absolute deviation for 207 runs was 0.038.

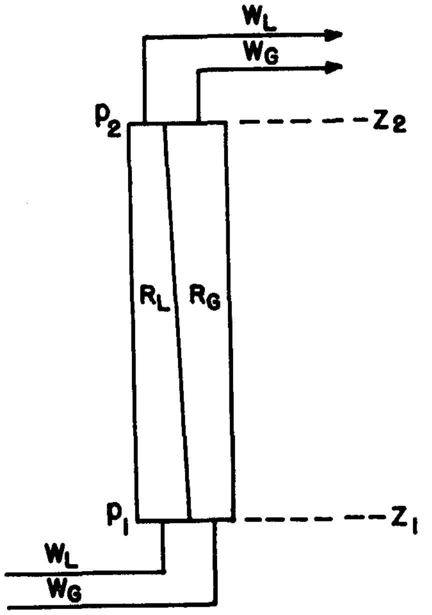

Fig. 4. Diagram of vertical concurrent two-phase flow.

Excellent agreement is obtained with the proposed correlation over a wide range of fluid physical properties and for pipe diameters from 0.4 to 2.34 in. in diameter.

ENERGY BALANCE FOR TWO-PHASE VERTICAL FLOW

As shown schematically in Figure 4 a known mass of liquid W_L and gas W_G at a pressure P enters the pipe section per unit time. The volume per unit time and from this the volumetric fraction of each phase at the inlet conditions can be calculated:

$$y_L = \frac{W_L v_L}{W_L v_L + W_G v_G}$$

Inside the section the gas travels at a higher velocity than the liquid. As a result the gas volume fraction in the pipe R_G is less than y_G, and the liquid volume fraction in the pipe $R_L = 1 - R_G$ is greater than y_L.

A mechanical energy balance can be made for the section shown by Figure 4. With the assumption that steady state exists, W_L pounds of liquid and W_G pounds of gas enter at 1 and leave at 2 in a given time interval. The mechanical energy balance form of the Bernoulli theorem is written as

$$\frac{g}{g_c} Z_1 + \frac{V_1^2}{2g_c} - w_s - \int_{p_1}^{p_2} v\,dp - F = \frac{g}{g_c} Z_2 + \frac{V_2^2}{2g_c} \qquad (3)$$

This equation applies to single-phase flow; for liquid-gas flow the equation can be applied separately to each phase. When W_L pounds of liquid and W_G pounds of gas enter per unit time, for the liquid phase

$$W_L \frac{g}{g_c} Z_1 + W_L \frac{V_{L_1}^2}{2g_c} - W_L w_{s_L} - W_L \int_{p_1}^{p_2} v_L\,dp - W_L F_L = W_L \frac{g}{g_c} Z_2 + W_L \frac{V_{L_2}^2}{2g_c} \qquad (4)$$

For the gas phase

$$W_G \frac{g}{g_c} Z_1 + W_G \frac{V_{G_1}^2}{2g_c} - W_G w_{s_G} - W_G \int_{p_1}^{p_2} v_G\,dp - W_G F_G = W_G \frac{g}{g_c} Z_2 + W_G \frac{V_{G_2}^2}{2g_c} \qquad (5)$$

Since v_L can be assumed constant and v_G continually increases almost linearly as p decreases, $-\int_{p_1}^{p_2} v_L\,dp$ reduced to $-v_L(p_2 - p_1)$ and $-v_{Gam}(p_2 - p_1)$ is a good approximation of $-\int_{p_1}^{p_2} v_G\,dp$. Work is neither added nor subtracted from the total system, but one phase

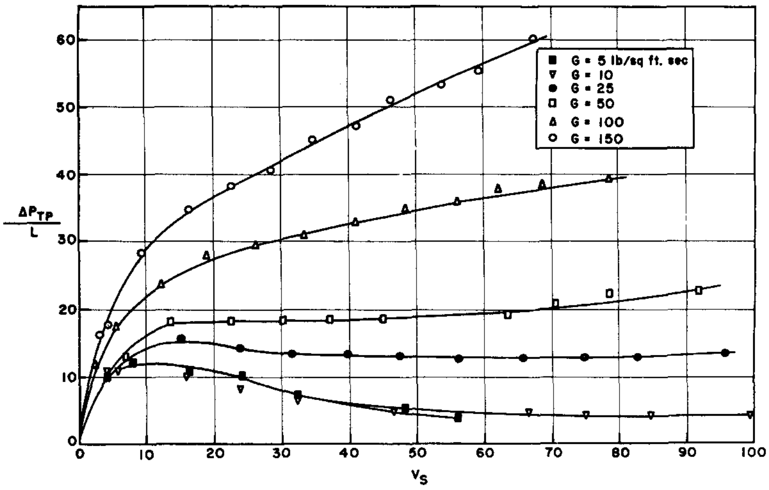

Fig. 5. Two-phase pressure drop vs. slip velocity for air-water data.

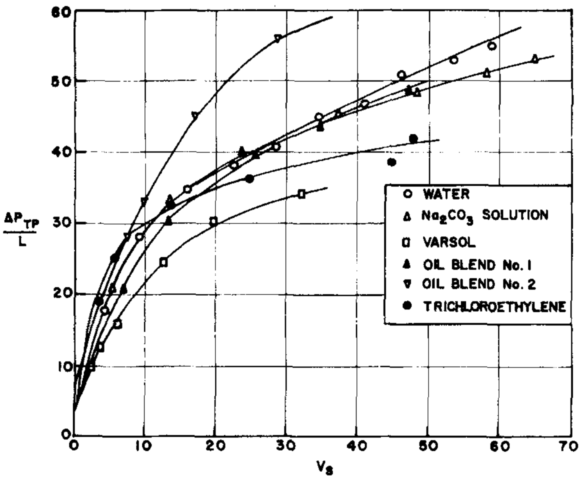

Fig. 6. Two-phase pressure drop vs. slip velocity for all systems at G = 150 lb./sq. ft. sec.

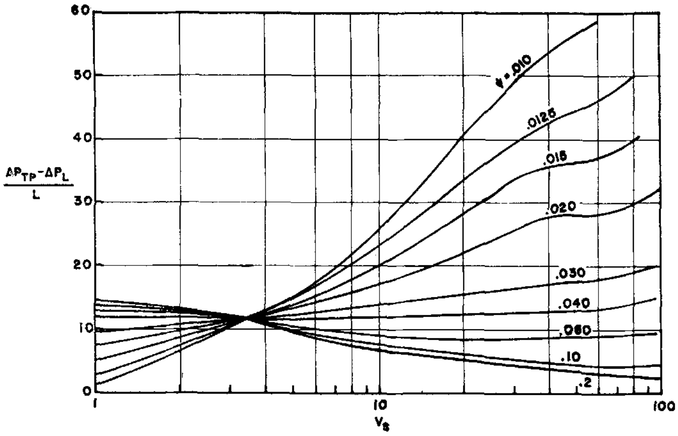

Fig. 7. Pressure-drop correlation.

may do work on the other. Therefore

$$-W_L w_{sL} = W_G w_{sG} \quad \text{or}$$

$$W_L w_{sL} + W_G w_{sG} = 0$$

Substituting and adding Equation (4) for the liquid and (5) for the gas phase one obtains

$$(W_L + W_G)\frac{g}{g_c}(Z_2 - Z_1) + \frac{W_L(V_{L_2}^2 - V_{L_1}^2)}{2g_c} + \frac{W_G(V_{G_2}^2 - V_{G_1}^2)}{2g_c} + W_L F_L + W_G F_G - (W_L v_L + W_G v_{Gam})(p_1 - p_2) = 0 \quad (6)$$

Assuming that $g/g_c = 1$ and dividing Equation (6) through by $(W_L v_L + W_G v_{Gam})$ one gets

$$\frac{(W_L + W_G)(Z_2 - Z_1)}{(W_L v_L + W_G v_{Gam})} + \frac{W_L(V_{L_2}^2 - V_{L_1}^2) + W_G(V_{G_2}^2 - V_{G_1}^2)}{2g_c(W_L v_L + W_G v_{Gam})} + \frac{W_L F_L + W_G F_G}{(W_L v_L + W_G v_{Gam})} - (p_1 - p_2) = 0 \quad (7)$$

Then defining

$$\Delta P_{TP} = \frac{W_L F_L + W_G F_G}{(W_L v_L + W_G v_{Gam})}$$

one obtains the following general equation for liquid-gas two-phase flow:

$$\frac{(W_L + W_G)(Z_2 - Z_1)}{(W_L v_L + W_G v_{Gam})} + \frac{W_L(V_{L_2}^2 - V_{L_1}^2) + W_G(V_{G_2}^2 - V_{G_1}^2)}{2g_c(W_L v_L + W_G v_{Gam})} + \Delta P_{TP} - (p_1 - p_2) = 0 \quad (8)$$

Equation (8) as derived here was used to interpret the new data obtained for vertical liquid-gas two-phase flow.

TABLE 3. SUMMARY OF COEFFICIENTS AND STANDARD DEVIATIONS FOR PRESSURE DROP CORRELATION

Physical property	Coefficient	Standard deviation
μ_L	−0.1476	0.019
σ	−0.1944	0.057
G	−0.6973	0.033

TWO-PHASE PRESSURE DROP

The second term of Equation (8) was found to represent a maximum of 1% of the total pressure difference. The kinetic energy calculated by a term similar to that suggested by Martinelli and Nelson (*15*) represents a maximum of 1.5% of the total pressure difference. Thus the kinetic energy term can be assumed to be negligible, and Equation (8) reduces to

$$\frac{(Z_2 - Z_1)(W_L + W_G)}{(W_L v_L + W_G v_{Gam})} - (p_1 - p_2) + \Delta P_{TP} = 0 \quad (9)$$

Equation (9) was used to calculate ΔP_{TP} from the experimental data. The first term, the pressure change due to potential head, was calculated based on the air and liquid rates entering the pipe. The pressure difference $(p_1 - p_2)$ was that measured between the two pressure taps.

The *first term* of Equation (9) is used as the potential head term rather than $(Z_2 - Z_1)(R_L v_L + R_G v_G)$ as suggested by Lottes and Flinn (*13*) because:

1. Equation (9) can be derived from theory as shown.
2. This equation can be used to get a correlation for ΔP_{TP} from the experimental data that can then be used as a general method to calculate $p_1 - p_2$.
3. The difference between experimental values of $p_1 - p_2$ and a head term based on R_L often gives negative values of ΔP_{TP}. Positive values of ΔP_{TP} are always obtained with Equation (9).

The two-phase friction pressure drop is due to two mechanisms, both of which cause energy losses: friction between the fluid and the pipe wall, and turbulence between the two phases. The latter of these could be expected to be a function of the difference in velocity between the two phases which is defined as the *slip velocity*. Slip velocity appears preferable to the slip ratio used by Lottes and Flinn (*13*), since the difference involves absolute values and should logically correlate shear between the phases. Figure 5 shows the two-phase pressure drop per foot of pipe vs. slip velocity for the air-water runs; as expected the total mass flow is a parameter. Similar plots were obtained from the data for the other fluids studied. Figure 6 shows $\Delta P_{TP}/L$ vs. slip velocity for all systems for tests made at a total mass velocity of 150 lb./sq. ft. sec.

The ordinate at $V_s = 0$ is the pressure drop for 100% liquid flow. To facilitate correlation and use, the ordinate was revised to $(\Delta P_{TP} - \Delta P_L)/L$. After the data for all mass flow rates were replotted in this manner, a value of $(\Delta P_{TP} - \Delta P_L)/L$ was read for each

TABLE 4. COMPARISON OF OBSERVED AND CALCULATED TOTAL PRESSURE DROP FOR EXPERIMENTAL RUNS

System	No. of runs	Average % absolute error
Air-water	182	14.2
Air-Na_2CO_3 solution	113	10.5
Air-varsol	96	14.3
Air-oil blend No. 1	90	9.2
Air-oil blend No. 2	57	6.4
Air-trichloroethylene	25	8.4

Average absolute error of 563 runs = 11.4%.

TABLE 5. COMPARISON OF OBSERVED AND CALCULATED TOTAL PRESSURE DROP FOR LITERATURE DATA

Investigator	System	Pipe diameter, in.	No. of runs	Average % error
Govier et al. (6)	Air-water	1.025	27	10.4
Galegar et al. (5)	Air-water	2.0	24	14.5
Galegar et al. (5)	Air-water	0.52	15	8.3
Schwarz (18)	Steam-water	2.34	10	11

system at each total mass velocity and for a common slip velocity of 40 ft./sec. These values were correlated by

$$\log \frac{\Delta P_{TP} - \Delta P_L}{L} = \log a_0 + a_1 \log \rho_L + a_2 \log \mu_L + a_3 \log \sigma + a_4 \log G \qquad (10)$$

The computer was used to obtain the best least-squares fit in the same manner as for the holdup correlation. A correlation coefficient of 0.984 was obtained with $a_1 = 0$. This indicates that the correlation is independent of liquid density. The data are shown by Table 3. The coefficients are at least three times the standard deviation which indicates that each is very significant. The parameter to correlate physical properties and total mass velocity is therefore

$$\psi = \frac{1}{\mu_L^{0.147}\, \sigma^{0.194}\, G^{0.70}} \qquad (11)$$

Plots were then made of $(\Delta P_{TP} - \Delta P_L)/L$ vs. ψ at intervals of slip velocity of 20 ft./sec. Best values of $(\Delta P_{TP} - \Delta P_L)/L$ were then obtained and plotted with ψ as parameters. The final graph is shown by Figure 7. There is no obvious significance to the intersection of all curves at a common slip velocity.

Total liquid-gas pressure drop can then be calculated with Equation (9). The two-phase pressure drop is calculated by adding ΔP_L to $\Delta P_{TP} - \Delta P_L$ obtained from the correlation for a specific fluid and total mass velocity which fixes the value of ψ. The proposed correlation procedure was checked with the experimental data by determining the percentage error between the observed and calculated total pressure drop. The results are tabulated in Table 4.

This correlation is based on a 1.049-in. pipe diameter with a relative roughness of 0.002. Published data are for smooth pipes of different diameters and were used to extend the correlation.

Galegar et al. (5) presented pressure drop for smooth pipes 2 in. and 0.52 in. in diameter. Govier et al. (6) presented data for a 1.025-in. diameter smooth pipe. These data were correlated with a diameter correction to ψ:

$$\psi = \frac{1}{G^{0.7}\, \mu_L^{0.147}\, \sigma^{0.194}} \left(\frac{D}{0.0873} \right)^{0.5} \qquad (12)$$

A correction for pipe roughness was made with a correction to Equation (9) to give the following equation for total pressure drop:

$$(p_1 - p_2) = \frac{(Z_2 - Z_1)(W_L + W_G)}{(W_L v_L + W_G v_{Gam})} + \frac{f_L}{(f_L)_{0.002}} \Delta P_{TP} \qquad (13)$$

The absolute average percentage error between the observed and calculated data are shown by Table 5.

Thus the correlation for total pressure drop in vertical two-phase flow appears to be accurate to within 15% for the range of data investigated.

SUMMARY

Correlations have been presented for holdup and pressure drop in isothermal two-phase concurrent upward flow in a vertical pipe. These avoid the necessity of knowing the exact flow regime. For holdup, liquid volume fraction in the pipe was correlated (Figure 3) with a function x:

$$x = \left(\frac{W_L}{W_G} \right)^{0.9} \frac{\mu_L^{0.19}\, \sigma^{0.205}\, \rho_G^{0.70}\, \mu_G^{2.75}}{G^{0.435}\, \rho_L^{0.72}}$$

The correlation shows an average absolute error of 0.025 for R_L over a wide range of gas and liquid physical properties. Holdup is independent of pipe diameter in the range of the available data, 0.4 to 2.34 in. The two-phase pressure drop is correlated graphically (Figure 7) as a function of the slip velocity with parameters of ψ based on the experimental data with the 1-in. pipe:

$$\psi = \frac{1}{G^{0.70}\, \mu_L^{0.147}\, \sigma^{0.194}}$$

Comparison with literature data for 0.5- to 2-in. pipe indicates that Ψ is proportional to the square root of the pipe diameter. Calculated total pressure drops show an average absolute error of about 11% from the experimental data.

The procedure developed for calculating the total pressure drop for vertical concurrent upward flow is as follows:

1. Calculate the pressu due to potential head, the firs the general equation:

$$\frac{(Z_2 - Z_1)(W_L + W_G)}{(W_L v_L + W_G v_{Gam})} - (p_1 - p_2) + \Delta P_{TP} = 0$$

2. Calculate the liquid volume fraction in the pipe with the holdup correlation for the fluid properties of the system.

3. Calculate the liquid and gas velocities in the pipe from the mass flows and 2. The difference between the liquid and gas velocities is the slip velocity.

4. Calculate ψ based on the total mass flow rate and the physical properties of the liquid. Read $(\Delta P_{TP} - \Delta P_L)/L$ from Figure 7 for the slip velocity from 3.

5. Calculate $\Delta P_L/L$ for the liquid flowing at the same total mass velocity of the system. $\Delta P_{TP}/L$ is the sum of this and the value used in 4.

6. $(p_1 - p_2)$ is then the sum of this ΔP_{TP} and the head term, based on the phase flow rates, as described by Equation (9).

NOTATION

a_0 = constant
a_1, a_2, a_3, a_4 = exponents
D = inside diameter of pipe, ft.
f = friction factor
$(f_L)_{0.002}$ = friction factor for liquid in pipe of relative roughness 0.002
G = mass velocity, lb. mass/sq. ft. sec.
g = acceleration of gravity, ft./sec.^{2}
g_c = 32.17 (lb. mass) (ft.)/(lb. force) (sec.^{2})
L = length of pipe, ft.
ΔP = pressure drop, lb. force/sq. ft.
p = absolute pressure, lb. force/sq. ft.
R = volume fraction in pipe
V = linear velocity, ft./sec.
v = specific volume, cu. ft./lb. mass
v_{Gam} = arithmetic mean of v_{G1} and v_{G2}, cu. ft./lb. mass
W = mass rate of flow, lb. mass/sec.
w_s = external work by fluid, ft.-lb. force/lb.-mass
X = parameter in Lockhart-Martinelli correlation
x = parameter in proposed holdup correlation
y = volume fraction entering the pipe
y_G = gas volume fraction, $1 - y_L$
Z = elevation, ft.

Greek Letters

Δ = finite change in variables
μ = centipoises

density, lb. mass/cu. ft.
= surface tension, dynes/cm.
= parameter in proposed pressure drop correlation

Subscripts

G = gas
G_{am} = gas-arithmetic mean
L = liquid
S = slip
TP = two phase
1 = point of inlet to section
2 = point of outlet to section

LITERATURE CITED

1. Chenoweth, J. M., and M. W. Martin, *Petrol. Refiner*, **34**, No. 10, p. 151 (1955).
2. Cromer, Sylvan, M. S. thesis, Univ. Oklahoma, Norman, Oklahoma (1937).
3. Dukler, Abraham E., M.S. thesis, Univ. Delaware, Newark, Delaware (1949).
4. Egen, R. A., D. A. Dingee, and J. W. Chastain, *Battelle Memorial Institute Rept. 1163* (1957).
5. Galegar, W. C., W. B. Stovall, and R. L. Huntington, *Petrol. Refiner*, **33**, No. 11, p. 208 (1954).
6. Govier, G. W., B. A. Radford, and J. S. C. Dunn, *Can. J. Chem. Eng.*, **35**, 58 (1957).
7. Isbin, H. S., N. C. Sher, and K. C. Eddy, *A.I.Ch.E. Journal*, **3**, 136 (1957).
8. Jenkins, Rodman, M.S. thesis, Univ. Delaware, Newark, Delaware (1947).
9. Jens, W. H., and P. A. Lottes, *Argonne National Laboratory Rept. 4627* (1951).
10. Kegel, Phillip K., B.S. thesis, Univ. Delaware, Newark, Delaware (1948).
11. Lockhart, R. W., and R. C. Martinelli, *Chem. Eng. Progr.*, **45**, No. 1, p. 39 (1949).
12. Lottes, P. A., *Proceedings 1955 Conference on Nuclear Engineering*, p. A-1.
13. ———, and W. S. Flinn, *Nuc. Sci. and Eng.*, **1**, p. 461 (1956).
14. McElwee, Frank D., M.S. thesis, Univ. California, Berkeley, California (1949).
15. Martinelli, R. C., and D. B. Nelson, *Trans. Am. Soc. Mech. Engrs.*, **70**, 695-702 (1948).
16. Radford, B. A., M.S. thesis, Univ. Alberta, Canada (1949).
17. Schurig, Wolfgang, *VDI-Forshungsheft*, **365**, 13 (1934).
18. Schwarz, Karl, *ibid.*, **445**, Edition B, p. 1 (1954).
19. Sher, N. C., *Westinghouse Atomic Power Division, TH-265* (1956).
20. ———, *Westinghouse Atomic Power Division, TH-300* (1957).
21. Yagi, Sakae, Takashi Shirai, and Teiji Sasaki, *Chem. Eng. (Japan)*, **15**, 317-322 (1951).
22. Yagi, Sakae, and Teiji Sasaki, *ibid.*, **17**, 216-223 (1953).

Manuscript received September 28, 1959; revision received June 14, 1961; paper accepted June 14, 1961. Paper presented at A.I.Ch.E. San Francisco meeting.

A Thermodynamic Correlation of Gas Solubilities

J. M. PRAUSNITZ and F. H. SHAIR
University of California, Berkeley, California

Low-pressure solubility data have been correlated for eleven gases in nine solvents over a wide temperature range by considering the dissolution process in two steps. First the gas is condensed isothermally to a hypothetical liquid at 1 atm. pressure, and then this hypothetical liquid is dissolved in the solvent. The free energy of the first step depends only on the properties of the solute, which in the case of nonpolar gases can be adequately described by the theorem of corresponding states. The second step depends on the properties of both solute and solvent and, in the case of nonpolar systems, can be described by the theory of regular solutions. The correlation depends on three solute parameters: the solubility parameter, the molar volume, and the fugacity of the hypothetical liquid; the last of these has been plotted as a generalized function for the reduced temperature range of 0.7 to 3.2. A separate plot is given for hydrogen. These parameters may be used to make good estimates of low-pressure gas solubilities (or K values) in nonpolar solvents over a wide range of temperature.

A semiempirical method for correlating the solubilities of gases in polar solvents is also described and illustrated for several cases.

Since the correlation presented in this paper covers a wide temperature range, it is possible to make estimates of the heats of solution of gases in liquids. These may be useful in enthalpy-balance calculations as required in certain phase-separation operations.

The chemical literature contains many data on the solubility of gases in liquids, but a critical evaluation of these data points to two serious deficiencies. First, many data are of doubtful accuracy as is evidenced by the fact that for the same system under identical conditions, two or more authors frequently report very widely differing results. Second, the vast majority of the data were obtained at 25°C., and data at less than 0°C. and higher than 60°C. are very scarce. It therefore appeared desirable to take reliable gas solubilities and correlate them as well as possible within a simple but useful thermodynamic framework with the aim that the correlation might be used to predict solubilities for systems not previously studied and for temperatures other than those near 25°C.

THERMODYNAMIC FRAMEWORK

A rigorous method for the prediction of gas solubilities requires a valid theory of solutions. Such a theory is, unfortunately, not available, but for a semiempirical description of nonpolar systems the theory of regular solutions can serve as the basis of a correlating scheme. Previous attempts to use regular solution theory for gas-liquid solutions *(4, 11, 28, 32, 33)* have been quite successful. However these earlier studies were concerned either with a particular class of solutions (such as hydrocarbons), or were limited to a particular temperature, or were chiefly interested in the theoretical aspects of the problem. In this work however the theory is used in as general a way as